Do People Choose in Accordance with Utility Theory?

A Re-run of Some Decision Experiments by Tversky/Kahneman and Others.

Klaus Manhart
mail@klaus-manhart.de

Institute for Advanced Studies, Vienna
Dept. of Mathematical Methods and Computer Science

1986

Do People Choose in Accordance with Utility Theory?

A Re-run of Some Decision Experiments by Tversky/Kahneman and Others.*)

Abstract

The Expected Utility Theory of von Neumann/Morgenstern is a widely accepted normative model of rational choice for risky situations. Beyond this, its descriptive validity keeps being postulated time and again. With reference to the experiments of Tversky/Kahneman and other authors, quite a number of important decision experiments have been remade, generally showing the same tendency as the original studies. Basic assumptions of the von Neumann/Morgenstern-Theory were systematically violated, since these assumptions ignore psychological principles that govern the perception of decision problems. The results mainly revealed two weak points of the model: people neither structure problems holistically nor treat information according to the assumptions of the model - especially not those of probability.

***)** I would like to thank Andreas Diekmann for his advice in the preparation of this work and Iain Paterson for proof-reading and worthwhile comments on an earlier draft of the paper.

1. Introduction

In quite a number of situations people are confronted with decisions of which the results are uncertain; they just have known or assumed probabilities of occurring. These risky decisions can be interpreted as choices between various lotteries. An individual lottery L can formally be described as

$$L = (p_1\ x_1;\ p_2\ x_2;\ \ldots;\ p_n\ x_n), \quad \text{where } \sum_{i=1}^{n} p_i = 1.$$

According to L, result x_1 has probability p_1, result x_2 probability p_2 .... result x_n probabilty p_n.

The criteria of decision rationality under risk are given by the prescriptive decision theory. It makes the attempt to formulate the requirements of a rational decision and gives directives concerning alternatives. Although the term "rationality" has frequently been discussed in a controversial manner a general agreement has been reached on the point that rational choices should meet some basic requirements.

The standard theory of rational choice is the Expected Utility (EU-) Theory founded by the mathematician John von Neumann and the economist Oskar Morgenstern. The model consists of a number of axioms that can be seen as requirements of rational behavior in risky situations. For example, two axioms request transitivity for outcomes and lotteries, the reduction axiom requests, that compound lotteries are regarded as equivalent to the reduced lottery which results from applying the rules of probability (more details of some axioms are given in the following chapters).[1] A sensible individual is expected to satisfy the axioms of the model. Assuming the axioms are satisfied, each individual's choices can be described in terms of utilities of various outcomes. The individual acts in order to maximize expected utility, which means:

(T1) A result x_i is preferred to a result x_j if and only if the utility of x_i is greater than the utility of x_j:
x_i **f** x_j iff $u(x_i) > u(x_j)$,
where we use '**f**' to denote the preference relation; notice the difference: '**f**' is a relation between outcomes (or lotteries), '>' the common mathematical relation between numbers.

[1] For a complete formal treatment see von Neumann/Morgenstern (1953), chap.3 and Luce/Raiffa (1964), chap.2.

(T2) The utitity of a lottery is equal to the expected utility of its outcomes:
$u((p_1\ x_1;\ p_2\ x_2;\ ...;\ p_n\ x_n)) = p_1 \cdot u(x_1) + p_2 \cdot u(x_2) + ... + p_n \cdot u(x_n)$

Not only has the EU-Theory generally been accepted as a normative model, it has always been claimed, that the model can be used as a description and explanation of empirical decision behavior. It is assumed, that all reasonable people would wish to obey the axioms of the theory and that most people actually do so.

A most relevant contribution to this question was given by the studies of Amos Tversky and Daniel Kahneman. Their empirical studies suggest, that the EU-Theory **cannot** be seen as an adequate descriptive model of actual decision behavior. Significant as well as systematic variations turned up in regular course in their studies.[2]

To re-run some important decision experiments of Tversky/Kahneman and others, 125 students of a Munich University Campus were chosen. Subjects were to decide between different alternatives. The results mostly were hypothetical amounts of money, given in DM. At the time of the study, the exchange rate was approximately 10 DM for $4. There were 1-, 2- and 3- factorial designs: some decision problems were presented to the whole population others were split up in two or three parts. 2- and 3-factorial designs were controlled by randomizing.

The overall number of respondents will be indicated by N in the following. For each lottery you find a percentage number in brackets indicating people favoring the respective alternative.

2. Variations of Framing

A problem can be described in many different ways. Different terms for instance may be used to describe a certain fact or situation. The EU-Theory assumes that decisions are never influenced by the way problems are described: a rational choice implies that preferences are not to be changed if the decision frame is altered.
Problem 1 and 2 illustrates the effect of changing the frame (Tversky/Kahneman 1981: 453):

---

[2] For a review of their empirical results and theoretical insights see Kahneman/Tversky (1982).

Problem 1 (N = 64)
Imagine that there is an outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:
A) If Program A is adopted, 200 people will be saved; ***(39.1%)***
B) If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved. ***(60.9%)***
Which of the two programs would you favor?

The second group was given the same problem in a different wording of alternatives:

Problem 2 (N = 55)
C) If Program C is adopted 400 people will die; ***(25.5%)***
D) If Program D is adopted, there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die. ***(74.5%)***
Which of the two programs would you favor?

Problem 1 and 2 are effectively identical. What makes the difference is, that in Problem 1 the alternatives are made up by the number of saved lives whereas in Problem 2 the alternatives refer to lives lost. Rational behavior would now require approximately the same distribution of answers within each Problem.

Although in both Problems a majority prefer to be risk seeking and thus prefer a risky "game" to the expected value more individuals chose to be risky in Problem 2 (appr.15%). The difference however does not seem to be that big and is not sufficient to conclude a **significant** violation of the preferences independent of description.

More significant results could be drawn from the original studies of Tversky/Kahneman: 72% chose A, 28% B, 22% C and 78% D. (N = 152 in Problem 1; N=155 in Problem 2). The distribution of responses in C and D approximately corresponds to our results: a majority prefered D to C, the rest chose the risky alternatives.
With Tversky/Kahneman, however, the preferences in Problem 1 reversed: a majority now chose the non-risky alternative A. This contradicts the assumption of rational choice in a more significant way than the results of our experiments.

Tversky/Kahneman explain their result with the variation of decision framing: the normal reference point in Problem 1 is the death of 600 individuals. Saved lives as the consequences of the program were seen as gain. The normal reference point in Problem 2 however was the total absence of cases of mortality. What was important now for the decision was the losses of lives (Kahneman/Tversky 1982: 140).

But seeing the same game at one time in terms of gains, the other in terms of losses, then a well-known psychological principle governs the decision: choices involving gains are often risk averse whereas choices involving losses are often risk taking (see Problem 8; we will observe this pattern also in the next gain-loss-experiments). Thus following this principle in this game can lead to paradoxical effects.

3. Isolation Effects: Ignoring Probabilities

According to Tversky/Kahneman inconsistent preferences and violations of the EU-assumptions frequently result from the fact that decision makers neglect certain components - components **shared** by decision alternatives. Decision makers however concentrate on those components that **distinguish** between the alternatives. Tversky/Kahneman call this the **isolation effect** (Kahneman/Tversky 1979: 271).

Three Problems now describe the isolation effect when objectively identical probabilities are presented in different ways. The three Problems were given to three different groups (Tversky/Kahneman 1981: 455):[3]

Problem 3 (N = 42)
Which of the following options do you prefer?
A) a sure win of 60 DM; ***(52.4%)***
B) 80% chance to win 90 DM. ***(47.6%)***

Problem 4 (N = 46)
Which of the following options do you prefer?
C) 25% chance to win 60 DM; ***(8.7%)***
D) 20% chance to win 90 DM. ***(91.3%)***

[3] In the original the amount of 60 DM was replaced by $30 - 90 DM by $45; a proband chosen by random (p = 0.1) could win the money mentioned above.

Problem 5 (N = 37)

Consider the following two-stage game. In the first stage, there is a 75% chance to end the game without winning anything, and a 25% chance to move into the second stage.

If you reach the second stage you have a choice between:

E) a sure win of 60 DM; ***(51.4%)***

F) 80% chance to win 90 DM. ***(48.6%)***

Your choice must be made before the game starts, i.e. before the outcome of the first stage is known. Please indicate the option you prefer.

In Problem 5 the respondents were to choose between a 0.25•1.00 = 0.25 chance to gain 60 DM and a 0.25•0.80 = 0.20 chance to gain 90 DM. Problem 5 thus presents the choice between the lotteries (0.25 60 DM) and (0.20 90 DM) which were exactly the alternatives of Problem 4. Problem 4 and 5 are thus proved to be identical and the reduction axiom claims that the same choice is to be made in both problems.

As a matter of fact however 51.4% chose E in Problem 5 - in Problem 4 just 8.7% chose the E equivalent alternative C. This is a significant violation of the reduction axiom and it moreover suggests the isolation effect: the respondents seem to neglect components that are shared by the alternatives (1. stage of Problem 5) - they merely regard components that distinguish between the alternatives.

This assumption is confirmed by a comparison of Problem 5 and Problem 3. Problem 5 just differs from Problem 3 by the introduction of stage 1. Assuming stage 2 is reached, Problem 5 is identical with Problem 3. In case the game is finished at stage 1 the decision does not influence the result. Choices A and B in Problem 3 have the same distribution of responses as corresponding choices E and F in Problem 5. This confirms the hypothesis that the respondents neglected stage 1 in Problem 5. People pretended to have already reached stage 2.

The table below shows a comparison of my study with the original experiments of Tversky/Kahneman as well as with the same experiment of Holler with economics students in Munich (Holler 1983: 626ff):[4]

[4] Like me Holler made no real payments.

Table 1: Distribution of responses - comparison of my experiments with those of Kahneman/Tversky and Holler.

		my study	Kahneman/ Tversky *)	Holler **)
Problem 3 *) N=77 **) N=144	A	52%	78%	41%
	B	48%	22%	59%
Problem 4 *) N=81 **) N=184	C	9%	42%	7%
	D	91%	58%	93%
Problem 5 *) N=85 **) N=176	E	51%	74%	41%
	F	49%	26%	59%

In all experiments there was no distinction between Problem 3 and 5 i.e. stage 1 in Problem 5 was neglected and was not integrated into the evaluation.

There was an evidence in all experiments that alternatives C and E resp. D and F were not chosen in the same frequency as postulated by the reduction axiom. In both my and the Tversky/Kahneman studies a reversal of the most frequent alternative could be observed - just a slight one in my own study. No similar effect could be found in Holler's experiment. Even with Holler there was a more frequent choice of E in Problem 5 than of C in Problem 4.

Those 3 studies altogether confirm the isolation effect in an uniform manner and violate the reduction axiom.

Two additional notes:

(1) It is thoroughly possible that varying percentage between the reference population in all 3 experiments is due to the fact that the reference population has not been homogenous in any case (Holler: exclusively economics students; Tversy/Kahneman and me: students of different fields). An additional reason maybe found in the different experimental condition. The relevant fact however was that equal conditions existed **within** the respective population.

(2) As for the risky behavior there seem to be a "Munich-effect". In both Munich experiments respondents showed a higher tendency to take risks than their counterparts in the Tversky/Kahneman study. Holler (1983: 627) suggests that risk taking behavior is obviously favoured by the absence of financial reinforcements.

4. Isolation Effects: Different Presentation of Outcomes

The following two problems show the isolation effect when the presentation of outcomes is different. Each Problem again was given to a different group (Kahneman/Tversky 1979: 273):[5]

Problem 6 (N = 59)
Assume, that in addition to whatever you own, you have been given 1.000 DM.
Now choose between
A) 50% chance to win 1.000 DM
50% chance to win nothing; ***(16.9%)***
B) a sure win of 500 DM. ***(83.1%)***

Problem 7 (N = 65)
Assume, that in addition to whatever you own, you have been given 2.000 DM.
Now choose between
C) 50% chance to <u>*lose*</u> *1.000 DM*
50% chance to lose nothing; ***(67.7%)***
D) a sure loss of 500 DM. ***(32.3%)***

Regarding the final payments, both problems again are identical.
The reason is that, integrating the bonus of 1.000 DM, Problem 6 is reduced to the alternatives
A') (0.50 2.000 DM; 0.50 1.000 DM)
B') (1.00 1.500 DM).
In the same manner Problem 7 is reduced to
C') (0.50 2.000 DM; 0.50 1.000 DM)
D') (1.00 1.500 DM)

Therefore there is no rational reason to prefer the risky game C at one time and the certain one B at another. The big majority, however, followed this - with respect to EU-Theory - irrational principle.

The respondents did obviously not integrate the bonus to compare the lotteries since both problems had a bonus in common. Moreover the preference patterns of Problem 6 and 7

[5] In the original the DM-amounts correspond to Israeli Pounds; there were no actual payments.

imply inconsistency with the EU-Theory: according to the theory the same utility is attributed to an amount of e.g. 100.000 DM, no matter whether it had been gained from an a priori amount of 95.000 DM or 105.000 DM. As a matter of consequence the choice between a total wealth of 100.000 DM and the lottery (0.50 95.000 DM; 0.50 105.000 DM) should not be dependent of whether one currently owns the smaller or larger of these two amounts (Kahneman/Tversky 1979: 273). In contrast, the results of the experiments with less money showed clearly that there is no empirical evidence for this.

Exactly the same distribution resulted from the experiment of Tversky/Kahneman: 16% chose A, 84% B, 69% C and 31% D (N=70 in Problem 6, N=68 in 7).

5. The Non-Integration of Combined Lotteries

The EU-Theory claims that the conjunction of two independent choices must comply with the demands of rationality as well. Preference orders must not reverse if alternatives are combined. The following problems were given to all subjects (Tversky/Kahneman 1981: 454):

Problem 8 (N = 124)
Imagine that you face the following pair of concurrent decisions. First examine both decisions, then indicate the options you prefer.
Decision 1:
Choose between
A) a sure gain of 240 DM; ***(71.8%)***
B) 25% chance to gain 1.000 DM,
75% chance to gain nothing. ***(28.2%)***
Decision 2:
Choose between
C) a sure loss of 750 DM; ***(12.2%)***
D) 75% chance to lose 1.000 DM,
25% chance to lose nothing. ***(87.8%)***

At first it can be stated that the majority choice is risk averse in the gain lottery (decision 1): a certain gain is preferred to a risky lottery with approximately the same expected value of money. In contrast to this fact the majority choice in the loss lottery (decision 2) is risk taking: a risky loss ticket is preferred to a certain loss with same expected value of money.

The frequently observed behavior to take risk in gain lotteries but to avoid risks in loss lotteries is called the **reflection effect** by Kahneman/Tverksy (1979: 268). This decision pattern could already be found in the above Problems 6 and 7, if the bonus is not integrated in problem evaluation.

Since in Problem 8 decision 1 and 2 were given together, the subjects were to choose between the combinations A&C, A&D, B&C and B&D. As response distribution of this combined choices resulted (N=122): A&C 9.8%, A&D 61.5%, B&C 2.5%, B&D 26.2%. The most frequently chosen combination A&D is still inferior to the least chosen combination B&C. This is because the preference of combination A&D to B&C means a preference of a lottery
A&D (0.25 240 DM; 0.75 –760 DM)
to a lottery
B&C (0.25 250 DM; 0.75 –750 DM).

Although the integration of decision 1 and 2 proved comparatively simple - as actually suggested by the given instruction - decisions weren't integrated this way.[6]

The Tversky/Kahneman experiments again showed the analog distribution pattern: 84% chose A, 16% B, 13% C and 87% D (N=150); the combination A&D was preferred by 73%, B&C by 3%.

As Tversky/Kahneman assume, each decision in Problem 8 is conceived as a separate choice: "The respondents apparently failed to entertain the possibility that the conjunction of two seemingly reasonable choices should lead to an untenable result." (Tversky/Kahneman 1981: 455).
In addition to this the authors suppose, that a number of real decisions are independent and preference orders reverse, if decision alternatives are combined.

As a control experiment, Problem 8 was chosen in order to test the influence of actual payments. It was given to a different population (all of them students again) which **really** could gain or lose money - but with less pay-offs than above (about 1/100 of the these pay-offs).

[6] As soon as the prevalence of B&C over A&D becomes evident all of the respondents chose B&C (100%), see Tversky/Kahneman (1981: 454):

Summing it up, no decisive changes could be found concerning the assumption, that actual payments could possibly modify decision behavior. This control experiment therefore doesn't support Holler's hypothesis mentioned above, that risk taking behavior is favored by the absence of financial reinforcements.

6. Overevaluation of Low Probabilities

The EU-Theory assumes that people treat objectively given probabilities in an "objective" way - their decision behavior is thought to conform accordingly. Various experiments, however, proved, that there is an overevaluation of low objective probabilities whereas high probabilities are frequently underevaluated - a violation of the EU-assumptions.
The following problems, given to different groups, illustrate the overevaluation of low probabilities (Kahneman/Tversky 1979: 281):[7]

Problem 9 (N = 66)
Choose between
A) 0.1% chance to gain 5.000 DM,
99.9% chance to gain nothing; ***(59.1%)***
B) a sure gain of 5 DM. ***(40.9%)***

Problem 10 (N = 58)
Choose between
C) 0.1% chance to <u>loose</u> 5.000 DM,
99.9% chance to loose nothing; ***(48.3%)***
D) a sure loss of 5 DM. ***(51.7%)***

In Problem 9 the majority preferred a ticket of low probabilities to the certainly expected value of money. In Problem 10 still over 50% preferred a small certain loss to a minimal probability of a big loss.

As mentioned above, an often observed decision behavior is to choose risk averse in gain lotteries and risk taking in loss lotteries. According to this "reflection effect", which appeared also in the above given gain-loss-experiments - it could be expected that the majority would choose risk averse in Problem 9 which means to prefer B to A, and risk averse in Problem 10 with C to D. The empirical observed decision behavior however is

[7] DM-amounts correspond to Israeli Pounds, no actual payments.

now opposite and shows a slight reversal of the reflection effect. This can psychologically be interpreted that people give small probabilities high weighting: probabilities almost equal to null are apparently considered more important than they "objectively" are.

Problem 10 even provides a lottery model for insurances: a good half of respondents is willing to pay a low insurance tax in order to get rid of the low probability of a high loss. As soon as people give assurance against a comparatively rare accident, they obviously give high regard to extremely low probabilities.

As Tversky/Kahneman report there were even clearer cases of overevaluation of low probabilities: in their experiments 72% chose A, 28% B, 17% C and 83% D (N=72 in Problem 9 and 10).

7. Test of the Continuity of Mixed Lotteries

With reference to Schoemaker (1982: 542), EU-axioms imply that a lottery with the result x_1 and x_m is to be between the results x_1 and x_m, regarding the preference order. The lottery (0.25 10 DM) for example ought to be between the certain results 10 DM and 0 DM. Generalized, a mixed lottery (p L; (1-p) L') should have an attractiveness level intermediate to those of L and L'. Thus, regarding the 3 lotteries

A) (0.5 50 DM; 0.5 200 DM)

B) (0.5 100 DM; 0.5 150 DM)

C) (0.5 A; 0.5 B)

C should be in-between A and B in terms of attractiveness.

Although this is intuitively evident, Schoemaker fails to deduce it formally. To prove it, we use continuity and monotonicity.

Due to the continuity axiom there exist probabilities p, which can be combined with the best outcome x_1 and the worst outcome x_3 such that the lottery $(p\ x_1; (1-p)\ x_3)$ is as attractive as receiving the intermediate outcome x_2 for certain. Monotonicity assumes that for all probabilities p, p' and $x_1\ \mathbf{f}\ x_2$ holds:

$(p\ x_1; (1-p)\ x_2)\ \mathbf{f}\ (p'\ x_1; (1-p')\ x_2)$ iff $p > p'$.

Assuming, we have two outcomes x_1 and x_2 - which could also be lotteries A and B as above - 2 preference cases may now occur, if we do not allow indifference:

1) $x_1\ \mathbf{f}\ x_2$

2) $x_2\ \mathbf{f}\ x_1$.

case 1: $x_1 \succ x_2$, which can be written as $(1\ x_1; 0\ x_2) \succ (0\ x_1; 1\ x_2)$
According to continuity there exist probabilities which can be combined with x_1, x_2. Assuming $0 < p < 1$ is an arbitrary probability.
Then $(p\ x_1; (1\text{-}p)\ x_2)$ is a probability mixture and with monotonicity the preference order has to be:
$(1\ x_1; 0\ x_2) \succ (p\ x_1; (1\text{-}p)\ x_2)$.
Analog with monotonicity: $(p\ x_1; (1\text{-}p)\ x_2) \succ (0\ x_1; 1\ x_2)$
and therefore $(1\ x_1; 0\ x_2) \succ (p\ x_1; (1\text{-}p)\ x_2) \succ (0\ x_1; 1\ x_2)$.

case 2: $x_2 \succ x_1$. Analog to case 1 it can be proved:
$(1\ x_2; 0\ x_1) \succ (p\ x_1; (1\text{-}p)\ x_2) \succ (0\ x_2; 1\ x_1)$.

In both cases the lottery $(p\ x_1; (1\text{-}p)\ x_2)$ must neither be ranked in the first nor in the last place regarding preference. In particular a compound lottery C = (0.5 A; 0.5 B) must not be preferred before A or B.

With reference to Becker et al. (1963: 200) the following problem was given to all of the respondents:

Problem 11 (N = 125)
Choose between
A) 50% chance to gain 50 DM,
50% chance to gain 200 DM; ***(24.0%)***
B) 50% chance to gain 100 DM,
50% chance to gain 150 DM; ***(43.2%)***
C) 25% chance to gain 50 DM,
25% chance to gain 100 DM,
25% chance to gain 150 DM,
25% chance to gain 200 DM; ***(32.8%)***

As shown above, lottery C with (0.5 A; 0.5 B) is made up by A and B and ought not be preferred to A or B; exclusively the choice of A or B is compatible with the EU-Theory. As the results show as many as 1/3 of the respondents preferred alternative C and thus violate the EU-axiom.
Although this is a minority, it has another meaning than e.g. the same quantitative minority of approximately 30% in Problem 8. While in Problem 8 neither the choice of A nor B

necessarily violates the EU-assumptions - since individual utility functions are not known - the choice of C in Problem 11 does, regardless of the shape of a utility function.

The fact that the preference of C to A and B is incompatible with the EU-Theory is also proved by the deduction of the following contradiction (see also Schoemaker 1980: 20):

With C **f** A,
according to (T1) from the introduction: u(C) > u(A)
which means:
$u((0.25\ x_1; 0.25\ x_2; 0.25\ x_3; 0.25\ x_4)) > u((0.50\ x_1; 0.50\ x_4))$
and according to (T2)
(1) $0.25\ [\ u(x_1) + u(x_2) + u(x_3) + u(x_4)\ ] > 0.5\ [u(x_1) + u(x_4)]$.

In the same manner with C **f** B, according to (T1):
u(C) > u(B)
which means:
$u((0.25\ x_1; 0.25\ x_2; 0.25\ x_3; 0.25\ x_4)) > u((0.50\ x_2; 0.50\ x_3))$
and according to (T2)
(2) $0.25\ [\ u(x_1) + u(x_2) + u(x_3) + u(x_4)\ ] > 0.5\ [u(x_2) + u(x_3)]$.

Summing up (1) and (2) the result is contradictory:

$$\sum_{i=1}^{4} u(x_i) > \sum_{i=1}^{4} u(x_i).$$

In a test of Becker et al. 35 different questions in the manner of Problem 12 were given to 62 respondents: 60 subjects preferred alternative C at least once.

8. The Allais Paradox

The substitution axiom claims that the certain outcome x_i can be replaced in any situation through the lottery, for which indifference to x_i was measured by the continuity axiom. One of the first and best-known decision experiments in contradiction to the EU-Theory and in particular to the substitution axiom was made by Allais (1953: 20, see also Schoemaker (1980: 18) or Kahneman/Tversky (1979: 265f.):

Problem 12'

Choose between

A) *a sure win of 1.000.000 DM*

B) *10% chance to gain 5.000.000 DM,*
89% chance to gain 1.000.000 DM,
1% chance to gain nothing.

Problem 13'

Choose between

C) *11% chance to gain 1.000.000 DM,*
89% chance to gain nothing

D) *10% chance to gain 5.000.000 DM,*
90% chance to gain nothing

A predominant response pattern - even among decision experts - is the preference of A to B and D to C. Coombs et al. (1974: 154f.) and Savage (1954: 102) suppose that lottery A is preferred to lottery B, because nobody would like to miss the very chance to get wealthy. In the same way D is preferred to C by many people, because the great difference between the payments exceeds the small difference between the chances to win.

This seemingly harmless pair of preferences is, however, incompatible with the EU-Theory (see Schoemaker 1980: 18, or Coombs et al. 1974: 154):

From A **f** B in Problem 12' results with (T1)
u(A) > u(B), therefore
u((1.0 1 Mill DM)) > u((0.10 5-Mill-DM; 0.89 1-Mill-DM; 0.01 0-DM))
and with (T2):
u(1-Mill-DM) > 0.10·u(5-Mill-DM) + 0.89·u(1-Mill-DM) + 0.01·u(0-DM)
and therefore:
(1) 0.11·u(1-Mill-DM) > 0.10·u(5-Mill-DM) + 0.01·u(0-DM)

In the same way from D **f** C in Problem 13' results with (T1):
u(D) > u(C), therefore
u((0.10 5-Mill-DM; 0.90 0-DM)) > u((0.11 1-Mill-DM; 0.89 0))
and with (T2)
0.10·u(5-Mill-DM) + 0.90·u(0-DM)   > 0.11·u(1-Mill-DM) + 0.89·u(0)
and therefore:

(2) $0.10 \cdot u(\text{5-Mill-DM}) + 0.01 \cdot u(\text{0-DM}) > 0.11 \cdot u(\text{1-Mill-DM})$

Inequality (2) however contradicts unequation (1).

The fact, that the choices A to B and D to C contradict the substitution axiom can easily be seen if realizing the game as a lottery with 100 numerical tickets, one of which is drawn. It was Savage (1954: 103) who had this idea. Table 2 gives the Allais-Paradox matrix in the way of this lottery.

Table 2: The matrix of the Allais paradox according to Savage (outcomes in Million DM).

		ticket-number		
		1	2-11	12-100
Problem 12'	A	1	1	1
	B	0	5	1
Problem 13'	C	1	1	0
	D	0	5	0

The substitution axiom postulates that preferences never change if in either problem identical components are replaced by respectively different identical components. In the matrix above however the pay-offs off 1 Million DM (Problem 12') are replaced by payments of 0 DM (Problem 13') in the ticket numbers 12-100.

As soon as one of the tickets 12-100 is drawn it does not matter which alternative has been chosen. The decision maker therefore should be guided only by the consequences of the ticket numbers 1-11. In this case, however, both problems are identical which means that a preference of A to B ought to cause the choice C to D and vice versa, B to A with D to C.

Incompatible with the substitution axiom are choices A to B and D to C resp. vice versa B to A and C to D.

The first time confronted with the Allais-example, Savage also preferred the choice combination that was equally incompatible with the EU-Theory. He, however, reversed his decision when he realised the mistake. Savage's reaction illustrates in his view, how the theory can, in normative respect, be seen as a directive or means of correction for reasonable people.

In my own study the amount of money were reduced to a realistic level - so the Allais experiment was given to all respondents in the following way:

Problem 12 (N = 122)
Choose between
A) a sure win of 500 DM ***(20.0%)***
B) 10% chance to gain 2.500 DM,
89% chance to gain 500 DM,
1% chance to gain nothing. ***(80.0%)***

Problem 13 (N = 123)
Choose between
C) 11% chance to gain 500 DM,
89% chance to gain nothing. ***(0.0%)***
D) 10% chance to gain 2.500 DM,
90% chance to gain nothing. ***(100.0%)***

80.3% chose the alternative B&D, 19.7% A&D (N=122); the rest of the combinations B&C and A&C were not chosen at all.

The results are remarkable in two respects. At first it must be stated that in Problem 13 the respondents' preference order were identical with that predominate preference mentioned above. What is amazing in particular is that all (!) of the respondents preferred alternative D.

The second statement is, that in Problem 12 the big majority preferred alternative B. B preferred to A, however, does not correspond with the preference order mentioned in the literature: 80% chose the alternative combination corresponding with the substitution axiom whereas only 20% preferred the inconsistent combination A&D. The vast majority acted in accordance with the axiomatic demands; in this experiment therefore a violation could not be found at all.

The result pattern contradicts the EU-violations reported by Allais and others. For example Kahneman/Tversky (1979) received in a variation of the Allais experiment 82% choosing A, 83% choosing C and 61 % choosing the EU-incompatible combination A&D. A possible reason could be seen in the fact that in my experiment the pay-offs had been considerably reduced.

It seems in a way likely that it has been easier to choose an alternative implying the chance to leave without gains at amounts of 2.500 DM resp. 500 DM, rather than at amounts of 5 Million DM resp. 1 Million DM.

9. EU-Conformity and Mathematical Education

At the end of this section it is to be tested whether a mathematical-scientific education possibly influences the consistency of choice patterns: individuals educated analytically - so the hypothesis - are more likely to meet the demands of the EU-Theory.

In order to test this hypothesis the population was split up into two groups: group 1 was made up by students of mathematics, computer science, physics and engineering, group 2 by students less mathematically skilled, like students of arts, social sciences etc.[8]

The distribution of study fields was at follows (N=122):

9.8%	mathematician/computer scientists
6.6%	physicists
27.0%	engineers
13.9%	arts like philosophers, linguists, literature (without teacher)
12.3%	biologists/physicians
8.2%	economists
8,2%	teachers
7.4%	psychologists/social scientists
6.6%	architects

The first 3 categories belonged to group 1, the rest to group 2.
Table 3 shows the distribution of alternative preferences for each of the goups.

[8] The size of the sample allowed no further differentiation. It would be interesting to add a third category of students with a basic statistical education.

Table 3: Distribution of alternative preferences for mathematically trained students and less mathematically trained students

		group 1: mathematical skilled students		group 2: less mathematical skilled students	
Problem 1	A	26.1%	(6)	47.5%	(19)
	B	73.9%	(17)	52.5%	(21)
Problem 2	C	16.0%	(4)	28.6%	(8)
	D	84.0%	(21)	71.4%	(20)
Problem 3	A	53.3%	(8)	50.0%	(13)
	B	46.7%	(7)	50.0%	(13)
Problem 4	C	0.0%	(0)	14.8%	(4)
	D	100.0%	(19)	85.2%	(23)
Problem 5	E	31.6%	(6)	68.8%	(11)
	F	68.4%	(13)	31.2%	(5)
Problem 6	A	25.8%	(8)	7.7%	(2)
	B	74.2%	(23)	92.3%	(24)
Problem 7	C	68.2%	(15)	66.7%	(28)
	D	31.8%	(7)	33.3%	(14)
Problem 8	A	55.8%	(29)	82.6%	(57)
	B	44.2%	(23)	17.4%	(12)
	C	15.1%	(8)	10.4%	(7)
	D	84.9%	(45)	89.6%	(60)
Problem 8 combined	A&C	13.0%	(7)	7.7%	(5)
	A&D	44.4%	(24)	73.9%	(48)
	B&C	3.7%	(2)	1.5%	(1)
	B&D	38.9%	(21)	16.9%	(11)
Problem 9	A	36.0%	(9)	72.5%	(29)
	B	64.0%	(16)	27.5%	(11)

Problem 10	C	57.1%	(16)	39.3%	(11)
	D	42.9%	(12)	60.7%	(17)
Problem 11	A	24.5%	(13)	24.6%	(17)
	B	47.2%	(25)	(39.1%	(27)
	C	28.3%	(15)	36.3%	(25)
Problem 12	A	0.0%	(0)	32.8%	(22)
	B	100.0%	(50)	67.2%	(45)
Problem 13	C	0.0%	(0)	0.0%	(0)
	D	100.0%	(52)	100.0%	(58)
Problem 12/13 combined	A&C	0.0%	(0)	0.0%	(0)
	A&D	0.0%	(0)	24.1%	(14)
	B&C	0.0%	(0)	0.0%	(1)
	B&D	100.0%	(50)	75.9%	(44)

As expected, group 1 proved a more consistent choice behavior in all of the problems than group 2.

As for Problem 1 and 2 the deviation of equivalent choices A/C and B/D in group 1 just differed by 10%, in group 2 however it differed by 20%.

The isolation effect does not work that strong in Problem 3 to 5 in group 1: the difference of percentage between C and the compound lottery equivalent to C, E, is 32%, whereas in group 2 it is 54%. In the same way the isolation effect is not so significant in group 1 as in group 2, regarding Problem 6 and 7.

In Problem 8 44% in group 1 preferred the worst alternative combination A&D but 74% in group 2. In group 1 the percentage of people who consider low probabilities important is significantly lower in Problem 9 and 10 than in group 2.

In Problem 12 36% of group 2 preferred lottery C incompatible with the EU-Theory, in group 1 the percentage is insignificantly lower.

Regarding the last two problems all of the respondents in group 1 chose alternative B and D and thus were in full accordance with the EU-demands. In group 2 on the contrary just 1/4 of the persons chose A and D and thus acted contradictory to EU-Theory.

Summing up the results it can be stated that the percentage of those violating the rationality demands of EU-Theory is constantly lower in the group of students mathematically trained than with the non-mathematical group.

How can this be explained? A formal oriented education includes courses in probability theory and statistics, in which students are "drilled" to compute expected values when confronted with risky phenomena. Although people in general don't act like maximizing expected value and this principle also is not very useful in an one-shot game, applying expected value as a general decision rule doesn't violate EU-assumptions. Regarding Table 3 it seems, that mathematically trained people apply the expected value rule more frequent than the other group and thus don't choose against EU-assumptions. For example whereas in Problem 13 all students of the two groups chose the alternative with the higher expected value, in Problem 12 1/3 of the non-mathematical didn't follow this principle - and thus violate EU - but all mathematically trained people chose according to expected value.

10. Summary and Prospects

For the vast majority of respondents the experiment of Allais showed results that are in accordance with the assumptions of the EU-model. The other experiments instead showed results that could be interpreted as more or less significant violations of the EU-model. In particular a great many of the Tversky/Kahneman experiments showed similar results contradictory to EU-Theory, just like the original studies.

The results disclose two weak points of the theory, if interpreted as a descriptive model:
(1) People don't structure problems holistically, see isolation effect;
(2) They treat information, especially probabilities, not according to the EU-rules.

With reference to these results it can alltogether be doubted whether the EU-theory could or even should serve as a general descriptive model of decision under risk.
As a consequence of their empirical studies - the above experiments are just part of them - Tversky/Kahneman developed a more refined descriptive model, called Prospect Theory. Re: Prospect Theory see Kahneman/Tversky (1979). Besides, there are a number of different alternative models existing. For an overview see Schoemaker (1980).

References

Allais, M. (1953). Le Comportement de l` Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l` Ecole Americaine. *Econometrica,* 21, 503-546.

Becker, G.M., DeGroot M.H., & Marschak, J. (1963). An Experimental Study of Some Stochastic Models for Wagers. *Behavioral Science*, 8, 199-202.

Coombs, C.H., Dawes, R.M., & Tversky, A. (1974). *Mathematical Psychology*. Englewood Cliff: Prentice-Hall.

Holler, M. (1983). Do Economic Students Choose Rationally? A Research Note. *Social Science Information*, 22, 623-630.

Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk, *Econometrica*, 47, 263-291.

Kahneman, D., & Tversky, A. (1982). The Psychology of Preferences, *Scientific American*, 1, 136-142.

Luce, R.D., & Raiffa, H. (1964). *Games and Decisions*. 4.ed. New York: Wiley.

von Neumann, J., & Morgenstern, O. (1953). *Theory of Games and Economic Behavior*. 3.ed. Princeton: Princeton University Press.

Savage, L.J. (1954). *The Foundations of Statistic*s. New York: Wiley.

Schoemaker, P.J.H. (1980). *Experiments on Decision under Risk: The Expected Utility Hypothesis*, Boston: Nijhoff.

Schoemaker, P.J.H. (1982). The Expected Utility Model: Its Variants, Purposes, Evidence and Limitations. *Journal of Economic Literature*, 20, 529-563.

Tversky, A., & Kahneman, D. (1981). The Framing of Decisions and the Psychology of Choice. *Science*,, 211, 453-458.