Cooperation in 2- and N-Person Prisoner's Dilemma Games: A Simulation Study

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Abstract

Simulation studies in the context of Robert Axelrod's research on iterative prisoner's dilemma games focus nearly exclusively on the 2-player-version of the game. In contrast, this article reports results of a simulation with an iterated N-person prisoner's dilemma where group size N varies between 2 and 30. The simulation investigates the relative performance of conditional cooperative strategies with increasing group size. Results show that some „nice" strategies like „TIT FOR TAT" are relatively successful and robust even in larger groups and non-nice environments. However, this does not solve the cooperation problem. On the contrary, the relative success of some „nice" conditional cooperative strategies is paralleled by a rapid decline of cooperation in large groups.

1. The Cooperation Problem in 2-Person Prisoner's Dilemma

The prisoner's dilemma (PD) is known as the paradigm for a social situation, in which two individuals - or in general: two decision-makers - achieve a more unfavourable result on the condition of individual rational action than if they choose both cooparative. In terms of game theory: for each player exists two strategies C (cooperate) and D (defective), where the point of intersection of the dominated, „defective" strategies is a parato-inefficient equilibrium (Matrix 1). The decision for the cooparative choice on both sides yields a pareto-optimal pay-off, but this is not an equilibrium point, for every player has an incentive, to give up cooperation and to exploit his opponent.

In general, a 2-person PD is defined by:
(1) $T > R > P > S$
(2) $(T + S)/2 < R$
where T, R, P and S are the pay-offs according to Table 1. Table 1 shows the pay-off matrix used by Axelrod (1984).

Table 1: Matrix of the 2-Person Prisoner's Dilemma

Player 2

	C	D
C	3, 3 (R,R)	0, 5 (S,T)
D	5, 0 (T,S)	1, 1 (P,P)

Player 1 (C / D rows)

Supposing the absence of exogen given social norms, moral rules or binding contracts, the choice of D is the individually rational strategy. The pay-off (1,1) in the example of Table 1 is worse than the pareto-optimal, „collective rational" result (3,3) if both players choose C. The significance of the PD game results from the fact, that the core of numerous, very interesting social conflict situations corresponds to the type of a PD structure. Examples are social exchange, the situation of two individuals in Hobbes original state or the arms race between two nations.

A solution of the dilemma in a <u>single</u> game requires more or less restrictive additional assumptions (social norms, contracts etc.). However, in the case of an iterative, <u>repeated</u> played PD cooperative solutions without additional assumptions are theoretical consistent justified. According to the theory of super-games (Friedman 1971; Taylor 1976, Raub/Voss 1986) in infinite repeated PD-games conditional cooperative super game strategies do exist. A super game strategy is a decision rule which determines C- and D-choices in the course of the iterated game. A strategy is conditional if the decision for C or D in round i of the basic game (Matrix 1) depends on the decision of the other player in the previous rounds.

In the theory of super games the following two theorems can be proven (Friedman 1971; 1977; Raub/Voss 1986):

(i) In the set of unconditional strategies of the super game there exists only one equilibrium strategy: All D (which means: „choose D in every round").

(ii) The set of conditional strategies contains a (not empty) subset of strategies which can provide - in dependence of the strategy of the opponent - for every basic game a cooperative and pareto-optimal decision.

Theorem (ii) says, that a cooperative „solution" of the PD is possible, if the game is repeated infinitely. „Infinitely" can be interpreted as „the round in which the game ends is not known". This correspondes to the introduction of a probability to end the game or a discont factor. The discont factor can be interpreted as a measure for the value of the future. The greater this factor, the easier will be the production of cooperation.

There is one problem with Friedman's analytical „solution". The pool of available strategies is infinite and it remains an open question, how these strategies come off in competition with themselves and with other strategies. Axelrod (1984) used the method of computer simulation to give an (temporarily) answer to this question. He asked a number of experts in game theory and other subjects to submit PD strategies in form of programmed algorithms. Every strategy had to play an iterated PD against itself and all the other strategies. The goal was to get a maximum number of scores.

The results of Axelrod have come to be well-known in various disciplines. We sketch the results very briefly. In two tournaments with 16 resp. 63 strategies it emerged that the simplest program TIT FOR TAT won surprisingly the most points. The TIT FOR TAT (TFT) strategy by Anatol Rapoport uses only two rules: first, TFT is a nice strategy which means, it defects not first, second, TFT imitates exactly the decision of the opponent in the last round of a sequence. Therefore, a C choice by the opponent will be rewarded with C, a D choice will be punished with D. TFT is a conditional cooperative equilibrium strategy in the super game. Axelrod explains the win of TFT with four features, which he interprets as general rules to play PD-games successfully:

- Be nice, which means, don't be the first to defect;
- Be provocable, which means, react on a D choice with D;
- Be forgiving, which means, an effort to achieve peace (C after a sequence of D's) will be answered with C;
- Be simple, which means, be transparent and not too complicated.

More important than the win of TFT is, that in the tournaments all nice strategies attain more points than all non-nice strategies. The reason is, that non-nice strategies exploit nice strategies sometimes, but achieve less points with conditional cooperative strategies like TFT than the nice programs among themselves. Further, non-nice strategies steal themselves points by mutual D/D combination. The success of nice strategies can be commented by Rapoport: „in weakness is strength".

Axelrod's studies have produced a huge sequence of experiments with varying conditions (for example Coleman 1986; Donninger 1986; Schuessler 1986; for a summary see Axelrod/Dion 1989). All these studies refer to tournaments with dyadic matches, i.e. the iterated 2-person PD. However, our interest in the following is the competition between more than two players: the N-person PD.

2. The Cooperation Problem in N-Person Prisoner's Dilemma

The simulation experiments by Axelrod use the iterated 2-person PD. However, the analytic results of Friedman (1971) are more extensiv. Theorem (ii) of the last section - which claims the existence of a cooperative equilibrium of conditional strategies - refers not only to the special case N=2 but to the general N-person PD. Like in the 2-person PD the question is, whether conditional cooperative strategies are successful in the N-person PD and - in the positive case - which of these strategies are the most successful.

The relevance of the N-person PD derives from a wealth of interesting social situations, in which more than two persons interact simultanously. The most prominent example is the production of a collective good in a group of N persons - a situation modelled by Hardin (1971) as (one shot) N-PD. If the same group members take a decision about the alternative „contribution to the collective good" (C-choice) or „Freeriding" (D-choice) again and again, this situation can bei reconstructed as an iterated N-person PD.

Although the existence of conditional cooperative equilibrium strategies in the N-person PD is theoretically proven, the production and maintenance of equilibrium is more difficult in a surrounding of hostile strategies than in the 2-person PD. Above all the problem is that sanctions doesn't hit defective players specific like in the 2-person case, but also the cooperative group members. If a D choice in round i is repayed in round i+1 by a D choice,

the sanction hits not only the D chooser but also all those players which chose cooperative in round i. If they for their part have a tendency to repay D choices, the cooperation level will decline by a growing number of rounds. Only in groups with exclusive nice strategies cooperation will be maintained for the whole game. Since by increasing group size the probability of an encounter of exclusive nice strategies falls, the chances for a success of defective strategies rise. Simultanously the average pay-offs to the members of the tournament go down. For ALL D therefore a rise in the rank order must be expected to an increasing N

with decreasing avarage pay-offs. Simultanously cooperative strategies will go down. The following questions however are outstanding:

(1) How fast will be the expected rise of ALL D to an increasing N?

(2) Are there cooperative strategies with high chances of success in growing groups?

(3) Will the initial superiority of nice versus non-nice strategies in the 2-person PD be reversed by an increasing N?

The simulation experiments try to give an answer to these questions.

3. Design of the simulation study

The game matrix of the N-person PD is a generalisation of the matrix in Table 1 with the following pay-off function:

(2) $A_C = 3(x - 1)/(N-1)$

       $A_D = (5x + (N - x - 1))/(N - 1)$

where A_C is the pay-off to a C-player in round i , A_D the pay-off to a D-player in round i, N the number of players and x the number of C-decisions in a round. (2) reproduces the matrix of Table 1 in the case of N=2. The pay-off function (2) yields the pay-off matrix of Table 2 for the general N-person case.

Table 2: Matrix of the N-Person PD

Number of other players, which choose C

	0	1	2	3	...	N-2	N-1
C $x = 1$		$x = 2$	$x = 3$	$x = 4$	...		
Player j	0	$3/(N-1)$	$6/(N-1)$	$9/(N-1)$	...	$3(N-2)/(N-1)$	3
D	$x = 0$	$x = 1$	$x = 2$	$x = 3$	...		
	1	$(N+3)/(N-1)$	$(N+7)/(N-1)$	$(N+11)/N-1)$	...	$(5(N-2)+1)/(N-1)$	3

Like in the Axelrod simulations a sequence contains 200 rounds. To study the effects of group size N, 13 tournaments are implemented: N=2, 3, ..., 10, 13, 15, 20, 30. Since not all strategies are described in Axelrod (1980a, 1980b, 1984) comprehensible and some strategies require a lot of program code we restricted the programmed strategies to a subset of strategies from the tournament by Donninger (1986).

The transmission of the 2-person PD strategies to the N-person case calls for a definition of defection. For example, does it constitute for TFT in a N-PD game a defection, if in round i-1 only one opponent chooses D, or is it a defection when 20%, 40%,... of the other strategies decided for D? This situation requires for a aggregation rule, which assigns on the basic of individual decisions the feature „D" or „C" to the whole group. This means the introduction of a further parameter „tolerance level" which decides, how much D choices are necessary to recognize the decision of the whole group as non-nice. For each conditional strategy different versions of these tolerance levels are possible. First of all, we restricted all strategies to strong variants, which means: decisions of the opponents are defective if at least one member ot the other players chose D.

We have constructed a pool of 13 strategies. From this pool a combination of strategies of group size N was carried out randomly. Every combination of strategies played the PD game 200 times. For every N 20.000 draws are carried out. This means, that in 13 tournaments with 200 iterations 20.000 x 13 x 200 = 52 millions of N-PD games must be calculated.

The following strategies were in the pool:

(1) TFT starts with a cooperative choice in round 1 and thereafter does what the other players have done in the previous move.

(2) CHAMPION cooperates in the first 10 rounds and plays TFT for the next 15 rounds. After 25 rounds the program cooperates, except the following conditions are satisfied:
   - the other players defected in the previous round,
   - the other players cooperated less than 60% until now,
   - a random number between 0 and 1 is greater than the cooperation rate of the other players in the previous rounds;

(3) FRIEDMAN is a complete severe rule: it defects the whole game after a single defection of the other players. Else it plays C.

(4) SHUBIK starts with C, defects after the first defection ot the other players and increases with every defection of the others the number of the own defections with 1.

(5) JOSS plays principally TFT, but cooperates only with a probability of 90% after a cooperative move of the opponent.

(6) RANDOM plays C or D with equal probability and independent of the other players.

(7) EATHERLEY pays attention to the frequency of cooperation of the opponents until now. After the others chose D, EATHERLEY defects with the probability of the ratio between the number of the opponent's defections and the number of rounds.

(8) TESTER searches for unconditional cooperative players to exploit them; on the other side TESTER avoids defections, if the opponents can not be exploited. TESTER defects in the first move to test the reaction of the others. Do they defect, TESTER apologizes for defection with a C and plays for the rest of the game TFT. Otherwise it cooperates in the 2. and 3. move but defects in the following in every 2. move.

(9) TIT FOR TWO TATS (TF2T) is a forbearing variant of TFT and plays only D after two consecutive D's of the opponents.

(10) ALL D defects in every round independent of the choice of the others.

(11) ALL C cooperates in every round.

(12) TIT FOR TAT C (TFTC) by A. Diekmann was the winner in Donninger's simulation. It plays TFT, but every 10. move independent of the opponent two times C.

(13) FRANCE was the winner in Donninger's simulation with modified pay-off matrix. It plays in every 4, 7, 10, 13th... move D, in the other moves C.

In the following the term „D-choice of the opponents" means in the case of conditional strategies, that at least one of the others chose D. Analogously, a cooperation of the opponents is given if and only if all of the other players chose D.

4. Results of the simulations

Eight of the thirteen strategies - 2/3 of all - are nice and do not start with a defection. Let's first see the results of the nice strategies in comparison with non-nice one's. Table 3 and Figure 1 give a complete overview of the 13 tournament results in the appendix. Nice strategies are signed by „+", non nice by „-".

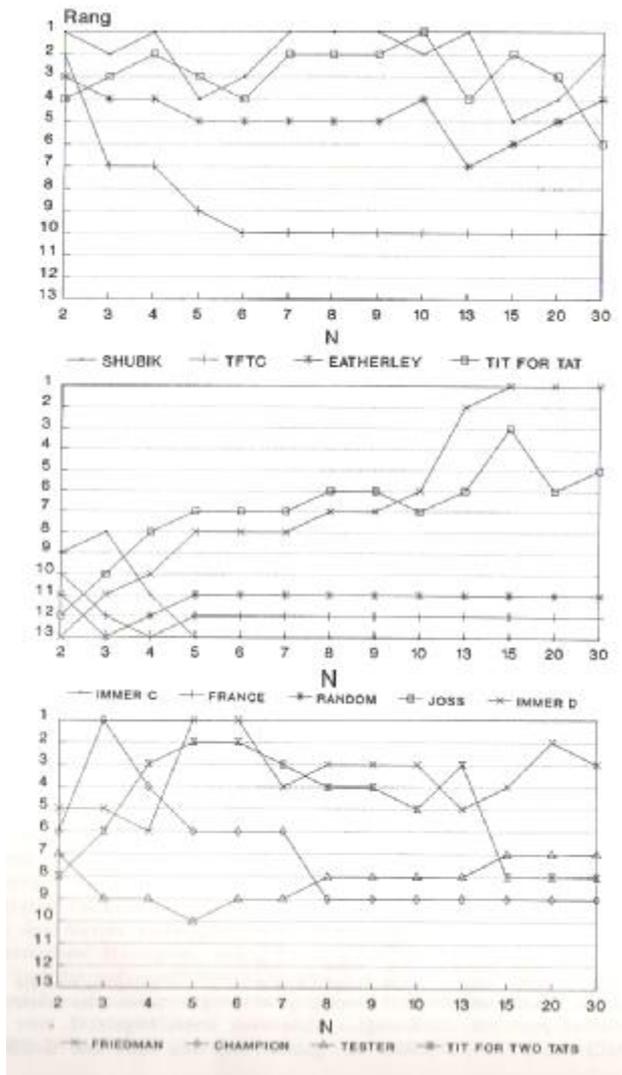Fig 1: Ranking of the strategies with varying group-sizes

Table 3: Results of the N-Person PD Computer Tournament with varying group sizes

RANG	N=2 STRATEGIE	PUNKTE	N=3 STRATEGIE	PUNKTE	N=4 STRATEGIE	PUNKTE	N=5 STRATEGIE	PUNKTE	N=6 STRATEGIE	PUNKTE
1	+ SHUBIK	2.76	CHAMPION	2.34	SHUBIK	2.09	FRIEDMAN	1.97	FRIEDMAN	1.90
2	+ TFTC	2.69	SHUBIK	2.32	TFT	2.09	TF2T	1.96	TF2T	1.89
3	+ EATHERLEY	2.68	TFT	2.28	TF2T	2.08	TFT	1.96	SHUBIK	1.89
4	+ TFT	2.62	EATHERLEY	2.28	CHAMPION	2.08	SHUBIK	1.96	TFT	1.89
5	+ FRIEDMAN	2.60	FRIEDMAN	2.26	EATHERLEY	2.06	EATHERLEY	1.93	EATHERLEY	1.87
6	+ CHAMPION	2.58	TF2T	2.24	FRIEDMAN	2.05	CHAMPION	1.91	CHAMPION	1.84
7	- TESTER	2.56	TFTC	2.19	TFTC	1.92	JOSS	1.78	JOSS	1.78
8	+ TF2T	2.54	IMMER C	2.05	JOSS	1.81	IMMER D	1.77	IMMER D	1.76
9	+ IMMER C	2.44	TESTER	1.98	TESTER	1.79	TFTC	1.77	TESTER	1.75
10	- FRANCE	2.31	JOSS	1.83	IMMER D	1.76	TESTER	1.76	TFTC	1.68
11	- RANDOM	2.23	IMMER D	1.77	IMMER C	1.41	RANDOM	1.20	RANDOM	1.18
12	- JOSS	2.11	FRANCE	1.51	RANDOM	1.25	FRANCE	1.02	FRANCE	0.98
13	- IMMER D	1.74	RANDOM	1.51	FRANCE	1.13	IMMER C	1.01	IMMER C	0.83

RANG	N=7 STRATEGIE	PUNKTE	N=8 STRATEGIE	PUNKTE	N=9 STRATEGIE	PUNKTE	N=10 STRATEGIE	PUNKTE	N=13 STRATEGIE	PUNKTE
1	SHUBIK	1.83	SHUBIK	1.81	SHUBIK	1.80	TFT	1.78	SHUBIK	1.77
2	TFT	1.83	TFT	1.81	TFT	1.80	SHUBIK	1.78	IMMER D	1.76
3	TF2T	1.83	FRIEDMAN	1.81	FRIEDMAN	1.79	FRIEDMAN	1.78	TF2T	1.76
4	FRIEDMAN	1.83	TF2T	1.80	TF2T	1.78	EATHERLEY	1.77	TFT	1.76
5	EATHERLEY	1.82	EATHERLEY	1.80	EATHERLEY	1.78	TF2T	1.77	FRIEDMAN	1.75
6	CHAMPION	1.76	JOSS	1.77	JOSS	1.77	IMMER D	1.77	JOSS	1.75
7	JOSS	1.76	IMMER D	1.76	IMMER D	1.77	JOSS	1.76	EATHERLEY	1.75
8	IMMER D	1.76	TESTER	1.75	TESTER	1.76	TESTER	1.75	TESTER	1.75
9	TESTER	1.75	CHAMPION	1.74	CHAMPION	1.73	CHAMPION	1.71	CHAMPION	1.69
10	TFTC	1.60	TFTC	1.56	TFTC	1.56	TFTC	1.55	TFTC	1.51
11	RANDOM	1.17	RANDOM	1.16	RANDOM	1.18	RANDOM	1.18	RANDOM	1.16
12	FRANCE	0.96	FRANCE	0.96	FRANCE	0.96	FRANCE	0.96	FRANCE	0.95
13	IMMER C	0.71	IMMER C	0.67	IMMER C	0.65	IMMER C	0.62	IMMER C	0.58

RANG	N=15 STRATEGIE	PUNKTE	N=20 STRATEGIE	PUNKTE	N=30 STRATEGIE	PUNKTE
1	IMMER D	1.76	IMMER D	1.75	IMMER D	1.76
2	TFT	1.76	FRIEDMAN	1.75	SHUBIK	1.75
3	JOSS	1.76	TFT	1.75	FRIEDMAN	1.75
4	FRIEDMAN	1.75	SHUBIK	1.75	EATHERLEY	1.75
5	SHUBIK	1.75	EATHERLEY	1.75	JOSS	1.75
6	EATHERLEY	1.75	JOSS	1.74	TFT	1.75
7	TESTER	1.75	TESTER	1.74	TESTER	1.75
8	TF2T	1.74	TF2T	1.74	TF2T	1.75
9	CHAMPION	1.69	CHAMPION	1.69	CHAMPION	1.69
10	TFTC	1.51	TFTC	1.50	TFTC	1.51
11	RANDOM	1.17	RANDOM	1.16	RANDOM	1.17
12	FRANCE	0.95	FRANCE	0.94	FRANCE	0.95
13	IMMER C	0.57	IMMER C	0.57	IMMER C	0.57

For N=2 the simulation corresponds to Axelrod's tournament with a restricted set of strategies. The results are similar to Axelrod. With the exception of TESTER nice and non-nice

strategies are perfectly divided: nice strategies take place on the first half of the table, non-nice one on the second part. However, with increasing group size the segregation pattern of the two sets of strategies will be mixed more and more. But the rank order of nice and non-nice strategies doesn't reverse completely in high group sizes. By increasing group size both nice and non-nice strategies have a chance to take higher scores. In all tournaments the average scores of the nice strategies exceeds the average scores of the non-nice strategies, whereas the overall pay-offs go down permanently (Fig. 2).

Fig 2: Average Pay-offs to friendly (top line) and unfriendly (bottom line) strategies sorted by group size
Pay-off
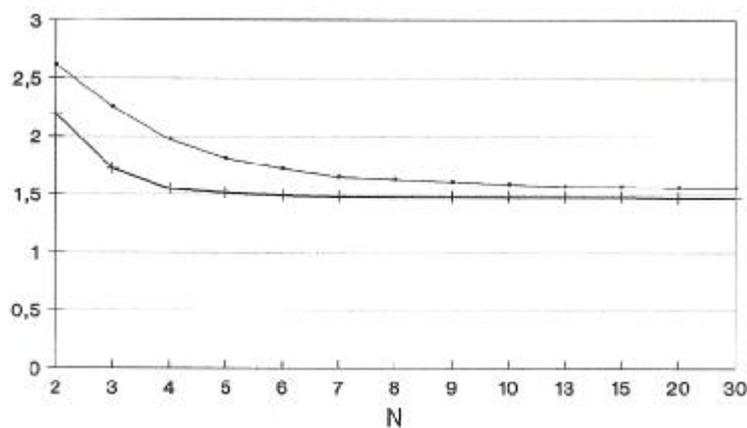
The constant decline of the average pay-off can be explained by the increase of defective reactions. In the conditional cooperative and uncooperative strategies the frequency of D/D combinations increases with group size. This can be clearly seen by regarding the pay-off for ALL C, which can achieve 3 scores only in the C/C combination. With a C/C profit of 2,44 for N=2 and 0,57 for N= 30 the share of C/C combinations decreases therefore from 81% (2,44/3) to 19%.

The rank order of ALL D improves on increasing group size, while ALL C continuously falls down (Fig. 1). But the rise of ALL D is not too quick. For middle sized groups until N=10 the defective strategy conquers only a rank order in the middle. In the case of N > 13 ALL D achieves the highest scores in comparison with the concurrents. However, in very large groups the simulation doesn't supply new findings which exceeds simple analytical reflections. If the group size exceeds the number of strategies to a high degree, then the defective strategy will be included in almost every randomly arranged group (f.e. for N=30 91%). But if practically all groups are infected by ALL D, then the success of a programm grows only by the number of defections.

What's about strategies with a higher tolerance level? In a new simulation all conditional strategies were implemented both in a „strong" and a „neutral" version. In the case of the strong version like before a group is classified as defective if at least one member of the opponents chooses D. In the new neutral version however at least 50% of D decisions are necessary to classify a group as defective. Therefore TFT-neutral reacts with D in a group with 7 opponents only if at least 4 of the others choose D. The results are listed in Table 4. In smaller groups the strong and neutral versions of conditional cooperative strategies are in front of the rank order. In big groups however the neutral versions have large losses because of the easy possibility to exploit them. The results of the remaining strategies don't change substantially in comparison to the former experiments.

Table 4: Simulation with „strong" („streng") and „neurtral" strategies

RANG	N=4 STRATEGIE	N=6 STRATEGIE	N=8 STRATEGIE	N=10 STRATEGIE	N=12 STRATEGIE	N=20 STRATEGIE	N=30 STRATEGIE
1	TFT NEUTRAL	FRIEDMAN NEUTRAL	FRIEDMAN STRENG	TFT STRENG	TF2T STRENG	TF2T STRENG	IMMER D
2	FRIEDMAN NEUTRAL	SHUBIK NEUTRAL	SHUBIK STRENG	FRIEDMAN STRENG	IMMER D	FRIEDMAN STRENG	JOSS STRENG
3	SHUBIK NEUTRAL	TFT NEUTRAL	FRIEDMAN NEUTRAL	SHUBIK STRENG	TESTER STRENG	IMMER D	TESTER STRENG
4	CHAMPION STRENG	TF2T STRENG	TF2T STRENG	TF2T STRENG	FRIEDMAN STRENG	SHUBIK STRENG	EATHERLEY STRENG
5	CHAMPION NEUTRAL	CHAMPION STRENG	SHUBIK NEUTRAL	TESTER STRENG	SHUBIK STRENG	TESTER STRENG	TFT STRENG
18	TESTER NEUTRAL	TFTC STRENG	TFTC NEUTRAL	TFTC NEUTRAL	TFTC NEUTRAL	TFTC NEUTRAL	TFTC NEUTRAL
19	JOSS STRENG	TESTER NEUTRAL	TESTER NEUTRAL	TESTER NEUTRAL	TESTER NEUTRAL	TESTER NEUTRAL	TESTER NEUTRAL
20	IMMER D	IMMER C	RANDOM	RANDOM	RANDOM	RANDOM	RANDOM
21	FRANCE	RANDOM	FRANCE	FRANCE	FRANCE	FRANCE	FRANCE
22	RANDOM	FRANCE	IMMER C	IMMER C	IMMER C	IMMER C	IMMER C

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